

USN

First Semester B.E. Degree Examination, Dec.2017/Jan.2018 Engineering Mathematics - I

Time: 3 hrs.

Max. Marks: 80

Note: Answer FIVE full questions, choosing one full question from each module.

Module-1

Find the nth derivative of $y = e^{-x} \sin x \cos 2x$. 1

(06 Marks)

Show that the curves $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$ cut each other orthogonally.

(05 Marks)

Find the radius of curvature of the curve $x^2y = a(x^2 + y^2)$ at the point (-2a, 2a). (05 Marks)

If $y = \sin(m\sin^{-1}x)$, then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$ (06 Marks) 2

Find the pedal equation of $r = 2(1 + \cos \theta)$. b.

(05 Marks)

Find the radius of curvature of $r^n = a^n \sin n\theta$.

(05 Marks)

Module-2

Expand $tan^{-1} x$ in powers of (x-1) upto the fourth degree term. 3

(06 Marks)

Evaluate $\lim_{x\to 0} \left[\frac{1}{x} - \frac{\log(1+x)}{x^2} \right]$

(05 Marks)

c. If z = f(x + ct) + g(x - ct), prove that $\frac{\partial^2 z}{\partial t^2} = C^2 \cdot \frac{\partial^2 z}{\partial x^2}$. (05 Marks)

Obtain the Maclaurin's series expansion of $e^{\sin x}$ upto the form containing x^4 . (06 Marks)

b. If $z = log\left(\frac{x^4 + y^4}{x + y}\right)$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial v} = 3$. (05 Marks)

c. If $u = x^2 + y^2 + z^2$, v = xy + yz + zx, w = x + y + z, show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 0$. (05 Marks)

Module-3

- a. A particle moves along the curve whose parametric equations are $x = t^3 + 1$, $y = t^2$ and z = 2t + 5. Find the components of its velocity and acceleration at time t = 1 in the direction of i + j + 3k.
 - b. If $\phi = 2x^3y^2z^4$, find Div(Grad ϕ).

(05 Marks)

c. Show that $\overrightarrow{F} = (y+z)i + (z+x)j + (x+y)k$ is irrotational. Also find a scalar function ϕ , such (05 Marks) that $\overrightarrow{F} = \nabla \phi$.

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- 6 a. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at P(1, -2, -1) in the direction of 2i j 2k. (06 Marks)
 - b. If $\vec{F} = (x+y+1)\vec{i} + \vec{j} (x+y)\vec{k}$. Show that \vec{F} curl $\vec{F} = 0$. (05 Marks)
 - c. If $\vec{F} = \nabla(xy^3z^2)$, find div \vec{F} and curl \vec{F} at the point (1, -1, 1). (05 Marks)

Module-4

- 7 a. Obtain the reduction formula for $\int \cos^n x dx$. (06 Marks)
 - b. Solve $ye^{xy}dx + (xe^{xy} + 2y)dy = 0$. (05 Marks)
 - c. Find the orthogonal trajectories of the family of curves $y^2 = Cx^3$. (05 Marks)

OR

- 8 a. Evaluate $\int_{0}^{1} x^{\frac{3}{2}} (1-x)^{\frac{3}{2}} dx$. (06 Marks)
 - b. Solve $\frac{dy}{dx} \frac{2}{x}y = \frac{y^2}{x^3}$. (05 Marks)
 - c. A body is heated to 110°C and placed in air at 10°C. After one hour its temperature becomes 60°C. How much additional time is required for it to cool to 30°C? (05 Marks)

9 a. Find the rank of the matrix $A = \begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$ (06 Marks)

- b. Solve the following system of equations by Gauss Jordan method: x+2y+z=3, 2x+3y+3z=10, 3x-y+2z=13 (05 Marks)
- c. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to the diagonal form. (05 Marks)

OR

10 a. Solve the following system of equations by Gauss-Seidal method: 20x + y - 2z = 17, 3x + 20y - z = -18, 2x - 3y + 20z = 25. Perform three iterations.

(06 Marks)

- b. Show that the transformation, $y_1 = 2x_1 2x_2 x_3$, $y_2 = -4x_1 + 5x_2 + 3x_3$, $y_3 = x_1 x_2 x_3$ is regular and find the inverse transformation.
- c. Reduce the quadratic form,

 $3x^2 + 3y^2 + 3z^2 + 2xy - 2yz + 2zx into the canonical form.$ (05 Marks)

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